# 1 Linear and Semilinear Set Definitions:

**Definition 1** *Let* $\mathbb{N}$ *be the set of nonnegative integers and* $k$ *be a positive integer. A set* $S \subseteq \mathbb{N}^k$ *is a* linear set *if* $\exists v_0, v_1, ..., v_t$ *in* $\mathbb{N}^k$ *such that*

$$S = \{v | v = v_0 + a_1 v_1 + ... + a_t v_t, a_i \in \mathbb{N}\}$$

*The vector* $v_0$ *(referred to as the* constant vector*) and* $v_1, v_2, ..., v_t$ *(referred to as the* periods*) are called the* generators *of the linear set* $S$*.*

**Definition 2** *A set* $S \subseteq \mathbb{N}^k$ *is* semilinear *if it is a finite union of linear sets.* $\emptyset$ *is a trivial semilinear set where the set of generators is empty.* Every finite subset of $\mathbb{N}^k$ is semilinear - *it is a finite union of linear sets whose generators are constant vectors. Clearly,* semilinear *sets are closed under union and projection. It is also know that semilinear sets are closed under intersection and complementation.*